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Normal Surfaces Associated with Adjoint Curves in E3

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Abstract
In this study, we work on the surfaces determined in relation to associated curves. We study normal surfaces defined with the help of adjoint curves, a special type of associated curve. For this, we first remember the basic equations of the 3-dimensional Euclidean space, which is the space we work with, and the adjoint curve issue. Then, by computing the first and second fundamental forms, principal curvatures, and mean and Gaussian curvatures of the normal surface of an adjoint curve, we obtain the characterizations of this surface and related some results.

1. Introduction

Surfaces and curves in differential geometry is a valuable topic that paves the way for studies in applied sciences by providing geometric expressions to fields such as physics, engineering and geophysics that serve technology. The subject of surfaces associated with curves, which we will discuss in this study, is one of the special examples of this. These surfaces, which are formed as a result of the motion of a curve or line depending on another curve, provide important conveniences in terms of giving geometric expressions to the subject [1-13].

Tangent, normal, and binormal surfaces, which are formed as a result of the motion of a curve in the direction of tangent, unit normal and binormal vector field due to the change of the time parameter, can be given as examples of surfaces associated with curves. Associated curves have an important place in determining the behavior and characterization of surfaces. The surfaces established with the help of adjoint curves, which is a type of associated curve, form the framework of our study [14-17].

In this study, we define the normal surfaces of adjoint curves and obtain the characterizations of these surfaces and some related results. First, we define the normal surface determining by the movement of the adjoint curve in the direction of the normal vector field. Then we obtain some results with the help of first and second fundamental forms, principal curvatures, Gaussian curvature, and mean curvature of this surface.

2. Preliminaries

In this part, we review the basic definitions and formulas related to the frame elements and the concept of adjoint curves that we have studied in the 3D Euclidean space. Next, we will discuss some fundamentals that have an important place in determining the behavior and characterization of a surface.

The Serret Frenet(SF) formulas in 3D Euclidean are given as

$$\begin{bmatrix} \nabla_s \boldsymbol{T} \\ \nabla_s \boldsymbol{N} \\ \nabla_s \boldsymbol{B} \end{bmatrix} = \begin{bmatrix} 0 & \mu & 0 \\ -\mu & 0 & \rho \\ 0 & -\rho & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{T} \\ \boldsymbol{N} \\ \boldsymbol{B} \end{bmatrix},$$

where μ, ρ are curvature and torsion of λ, respectively. Let s be the length of the arc [1]. Then, the SF frame formulas are given as

$$\boldsymbol{T} = \lambda'(s), \quad \boldsymbol{N} = \frac{\lambda''(s)}{\|\lambda''(s)\|}, \quad \boldsymbol{B} = \boldsymbol{T} \times \boldsymbol{N}.$$

Definition 1. Let $\{\boldsymbol{T}_\lambda, \boldsymbol{N}_\lambda, \boldsymbol{B}_\lambda\}$ be the SF frame of λ curve with s parameter. Then, the adjoint curve of λ according to the SF frame is defined as [2]

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$$\delta(s) = \int_{s_0}^{s} \boldsymbol{B}_\lambda(s)ds.$$

A surface that passes a straight line through each point is called a ruled surface. Surfaces such as cone, cylinder, helicoid, conical surface can be given as examples of ruled surfaces. A ruled surface is expressed as

$$\varphi(s,t) = \lambda + t\boldsymbol{X},$$

where the vector field X is the direction of the surface and the curve λ is the base curve of the surface [5].

Then we can construct a ruled surface whose base curve is curve λ and whose direction is the normal(unit) vector field of λ. This surface is called the normal surface defined by the curve λ. Hence, we give the definition:

Definition 2. The normal surface of a regular curve λ is defined by $\varphi(s,t) = \lambda + t\boldsymbol{N}$ [5].

Theorem 3. Let $\{\boldsymbol{T}_\lambda, \boldsymbol{N}_\lambda, \boldsymbol{B}_\lambda\}$ be the SF frame of λ curve with s parameter, δ be adjoint curve of λ according to the SF frame and μ_λ and ρ_λ be curvature and torsion of λ. Denote by $\{\boldsymbol{T}_\delta, \boldsymbol{N}_\delta, \boldsymbol{B}_\delta\}$ the SF frame elements for δ and denote by ρ_δ and μ_δ be torsion and curvature of δ. Then, the relationship between δ and λ can be given by the following equations [2]:

$$\boldsymbol{T}_\lambda = \boldsymbol{B}_\delta, \quad \boldsymbol{N}_\lambda = -\boldsymbol{N}_\delta, \quad \boldsymbol{B}_\lambda = \boldsymbol{T}_\delta,$$
$$\mu_\delta = \rho_\lambda \quad \rho_\delta = \mu_\lambda.$$

The normal(unit) vector field for any surface $\varphi(s,t)$ is defined by the equation

$$n = \frac{\varphi_s \wedge \varphi_t}{\|\varphi_s \wedge \varphi_t\|},$$

where $\varphi_t = \partial\varphi/\partial t$, $\varphi_s = \partial\varphi/\partial s$ and, t is parameter representing time. Then, the first and the second fundamental forms of φ are given by following equations:

$$\boldsymbol{I} = Eds^2 + 2Fdsdt + Gdt^2,$$
$$\boldsymbol{II} = eds^2 + 2fdsdt + gdt^2,$$

where

$$E = \langle\varphi_s, \varphi_s\rangle,\ F = \langle\varphi_s, \varphi_t\rangle,\ G = \langle\varphi_t, \varphi_t\rangle,$$
$$e = \langle\varphi_{ss}, n\rangle,\ f = \langle\varphi_{st}, n\rangle,\ g = \langle\varphi_{tt}, n\rangle. \quad (1)$$

Also, Gaussian and mean curvatures K and H are given as

$$H = \frac{Eg - 2Ff + Ge}{2(EG - F^2)},\ K = \frac{eg - f^2}{EG - F^2} \quad (2)$$

and principal curvatures are defined by [3-7]

$$k_1 = \sqrt{H^2 - K} + H,\ k_2 = H - \sqrt{H^2 - K}. \quad (3)$$

Theorem 4. A surface is minimal surface if and only if it has vanished mean curvature of this surface [1].

Theorem 5. A surface is a flat (developable) surface if and only if it has vanished Gaussian curvature of this surface [1].

Definition 6. If the relationship between the mean and Gaussian curvatures of a surface can be given by equation $H_sK_t - H_tK_s = 0$, then this surface is a Weingarten surface [12].

3. Normal Surfaces of Adjoint Curves

In this section, we discuss certain characterizations and results for the normal surface of an adjoint curve with the help of information given in the previous section.

Theorem 7. Let δ be adjoint curve of λ curve with arc length parameter. Then, first and second fundamental forms of normal surface of δ are given by the following equations:

$$\boldsymbol{I}_\delta = (t^2\mu_\lambda^2 + (1 - t\rho_\lambda)^2)ds^2 + dt^2,$$
$$\boldsymbol{II}_\delta = \frac{t\mu_\lambda'(1 - t\rho_\lambda) + t\mu_\lambda\rho_\lambda'}{\sqrt{t^2\mu_\lambda^2 + (1 - t\rho_\lambda)^2}}ds^2 + \frac{2\mu_\lambda}{\sqrt{t^2\mu_\lambda^2 + (1 - t\rho_\lambda)^2}}dsdt.$$

Proof. From the definition of normal surface, the normal surface of δ is written as

$$\varphi^\delta(s,t) = \delta + t\boldsymbol{N}_\delta.$$

As a result, the following equalities are obtained:

$$\varphi_s^\delta = t\mu_\lambda\boldsymbol{T}_\lambda + (1 - t\rho_\lambda)\boldsymbol{B}_\lambda,$$
$$\varphi_{ss}^\delta = t\mu_\lambda'\boldsymbol{T}_\lambda + (t\mu_\lambda^2 + t\rho_\lambda^2 - \rho_\lambda)\boldsymbol{N}_\lambda - t\rho_\lambda'\boldsymbol{B}_\lambda,$$
$$\varphi_t^\delta = -\boldsymbol{N}_\lambda, \quad \varphi_{tt}^\delta = 0, \quad \varphi_{st}^\delta = \mu_\lambda\boldsymbol{T}_\lambda - \rho_\lambda\boldsymbol{B}_\lambda.$$

and, from the equalities, unit standart normal vector field of φ^δ surface is found as

$$n_\delta = \frac{\varphi_s^\delta \times \varphi_t^\delta}{\|\varphi_s^\delta \times \varphi_t^\delta\|} = \frac{(1 - t\rho_\lambda)\boldsymbol{T}_\lambda - t\mu_\lambda\boldsymbol{B}_\lambda}{\sqrt{t^2\mu_\lambda^2 + (1 - t\rho_\lambda)^2}}.$$

These equalities are obtained similarly for the normal surface of λ curve. Then, with the help of Theorem 3 and of the equations given at the beginning of this section, we obtain

$$E = t^2\mu_\lambda^2 + (1 - t\rho_\lambda)^2, \qquad F = 0, \qquad G = 1,$$
$$e = \frac{t\mu_\lambda'(1 - t\rho_\lambda) + t\mu_\lambda\rho_\lambda'}{\sqrt{t^2\mu_\lambda^2 + (1 - t\rho_\lambda)^2}},\ f = \frac{\mu_\lambda}{\sqrt{t^2\mu_\lambda^2 + (1 - t\rho_\lambda)^2}},\ g = 0.$$

(4)

Hence, the first and the second fundamental forms of normal surfaces of δ are obtained as

$$\boldsymbol{I}_\delta = (t^2\mu_\lambda^2 + (1 - t\rho_\lambda)^2)ds^2 + dt^2,$$
$$\boldsymbol{II}_\delta = \frac{t\mu_\lambda'(1 - t\rho_\lambda) + t\mu_\lambda\rho_\lambda'}{\sqrt{t^2\mu_\lambda^2 + (1 - t\rho_\lambda)^2}}ds^2 + \frac{2\mu_\lambda}{\sqrt{t^2\mu_\lambda^2 + (1 - t\rho_\lambda)^2}}dsdt.$$

Corollary 8. Let δ be adjoint curve of λ curve with arc length parameter. Then, mean (H_δ) and Gaussian

(K_δ) curvatures of the normal surfaces of δ are given by the following equations:

$$K_\delta = \frac{-\mu_\lambda^2}{(t^2\mu_\lambda^2+(1-t\rho_\lambda)^2)^2}, \quad H_\delta = \frac{t\mu_\lambda'(1-t\rho_\lambda)+t\mu_\lambda\rho_\lambda'}{2\sqrt{(t^2\mu_\lambda^2+(1-t\rho_\lambda)^2)^3}}. \tag{5}$$

Proof. Using equations (4), we obtain

$$K_\delta = \frac{e_\delta g_\delta - f_\delta^2}{E_\delta G_\delta - F_\delta^2} = \frac{-\mu_\lambda^2}{(t^2\mu_\lambda^2+(1-t\rho_\lambda)^2)^2},$$

$$H_\delta = \frac{E_\delta g_\delta - 2F_\delta f_\delta + G_\delta e_\delta}{2(E_\delta G_\delta - F_\delta^2)} = \frac{t\mu_\lambda'(1-t\rho_\lambda)+t\mu_\lambda\rho_\lambda'}{2\sqrt{(t^2\mu_\lambda^2+(1-t\rho_\lambda)^2)^3}}.$$

Theorem 9. Let δ be adjoint curve of λ curve with arc length parameter. Then, the normal surface of δ is minimal if and only if

$$\frac{\mu_\delta'}{\mu_\lambda'} = \frac{t\mu_\delta - 1}{\mu_\lambda}.$$

Proof. From Theorem 3, $H_\delta = 0$. Therefore, we obtain $t\mu_\lambda'(1-t\rho_\lambda)+t\mu_\lambda\rho_\lambda' = 0$ with the help of (5). Using $\rho_\lambda = \mu_\delta$, the following result is obtained:

$$\frac{\mu_\delta'}{\mu_\lambda'} = \frac{t\mu_\delta - 1}{\mu_\lambda}.$$

We can easily obtain the following results with the help of Corollary 8:

Corollary 10. Let δ be adjoint curve of λ curve with arc length parameter. Then, principal curvatures of normal surface of δ are given by

$$k_{\delta_1} = [\sqrt{(\mu_\lambda'(1-t\rho_\lambda)+t\mu_\lambda\rho_\lambda')^2 + 4\mu_\lambda^2\rho_\lambda^2(\mu_\lambda^2+(1-t\rho_\lambda)^2)} + 2\mu_\lambda'(1-t\rho_\lambda) + 2t\mu_\lambda\rho_\lambda']/\sqrt{4(\mu_\lambda^2+(1-t\rho_\lambda)^2)^3},$$

$$k_{\delta_2} = [-\sqrt{(\mu_\lambda'(1-t\rho_\lambda)+t\mu_\lambda\rho_\lambda')^2 + 4\mu_\lambda^2\rho_\lambda^2(\mu_\lambda^2+(1-t\rho_\lambda)^2)} + 2\mu_\lambda'(1-t\rho_\lambda) + 2t\mu_\lambda\rho_\lambda']/\sqrt{4(\mu_\lambda^2+(1-t\rho_\lambda)^2)^3}.$$

Corollary 11. Let δ be adjoint curve of λ curve with arc length parameter. If the curvature and the torsion of δ is constant, then principal curvatures of normal surface of δ are given by

$$k_{\delta_1} = \frac{\mu_\lambda}{t^2\mu_\lambda^2+(1-t\rho_\lambda)^2}, \quad k_{\delta_2} = \frac{-\mu_\lambda}{t^2\mu_\lambda^2+(1-t\rho_\lambda)^2}.$$

Theorem 12. Let δ be adjoint curve of curve λ with arc length parameter. Then, the normal surface of δ is a flat surface if and only if it has vanished the curvature of the curve λ.

Proof. From Theorem 5 and equations (5), the proof is easily obtained.

Corollary 13. Let δ be adjoint curve of arc length parametrised curve λ. In this case, the following conditions are provided:

i) Let the curvature of λ be a nonzero constant. Then, the normal surface of δ is minimal if and only if the torsion of curve λ is a constant,

ii) Let be the curvature of λ isn't constant and the torsion of λ be constant. Then, the normal surface of δ is minimal if and only if the torsion of δ satisfies

$$\rho_\lambda = \frac{1}{t},$$

iii) If the normal surface of δ is a flat surface, then this surface is minimal.

Proof. From Theorem 4, Theorem 5, equations (5), all 3 cases claimed are plainly achieved.

Theorem 14. Let δ be adjoint curve of curve λ with arc length parameter. Then, the normal surface of δ is a Weingarten surface if and only if

$$\begin{aligned}&3\mu_\lambda\rho_\lambda^2(t\rho_\lambda-1)[(\mu_\lambda'(t\rho_\lambda-1)-t\mu_\lambda\rho_\lambda')(\mu_\lambda\mu_\lambda'-t\rho_\lambda'\\&+t^2\rho_\lambda\rho_\lambda')+(\mu_\lambda^2+(1-t\rho_\lambda)^2)(\mu_\lambda''+t\mu_\lambda\rho_\lambda''+t\rho_\lambda\mu_\lambda'')]\\&=(\mu_\lambda^2+(1-t\rho_\lambda)^2)^4[(\mu_\lambda\rho_\lambda'-\mu_\lambda'\rho_\lambda)(\mu_\lambda^2+(1-t\rho_\lambda)^2)\\&-3(\rho_\lambda-t\rho_\lambda^2)(\mu_\lambda'(1-t\rho_\lambda)-t\mu_\lambda\rho_\lambda')](\mu_\lambda^2\mu_\lambda'\rho_\lambda-\mu_\lambda'\rho_\lambda\\&-\mu_\lambda^3\rho_\lambda'-\mu_\lambda\rho_\lambda'-t^2\rho_\lambda^3\mu_\lambda'+t^2\rho_\lambda^2\mu_\lambda\rho_\lambda'+2t\rho_\lambda^2\mu_\lambda')).\end{aligned}$$

Proof. From Definition 2, if the normal surface of δ is a Weingarten surface, then

$$(H_\delta)_s(K_\delta)_t - (H_\delta)_t(K_\delta)_s = 0. \tag{6}$$

The partial derivatives of the Gaussian and mean curvatures of the normal surface of δ according to the s and t parameters are obtained as

$$\begin{aligned}(H_\delta)_t &= [2(\mu_\lambda\rho_\lambda'-\mu_\lambda'\rho_\lambda)(\mu_\lambda^2+(1-t\rho_\lambda)^2)^3\\&-6(\rho_\lambda-t\rho_\lambda^2)(\mu_\lambda^2+(1-t\rho_\lambda)^2)^2(\mu_\lambda'(1\\&-t\rho_\lambda)-t\mu_\lambda\rho_\lambda')]/[4(\mu_\lambda^2+(1-t\rho_\lambda)^2)^{\frac{9}{2}}],\\(H_\delta)_s &= [3(\mu_\lambda'(t\rho_\lambda-1)-t\mu_\lambda\rho_\lambda')(t^2\rho_\lambda\rho_\lambda'\\&-t\rho_\lambda'+\mu_\lambda\mu_\lambda')+3(\mu_\lambda^2+(1-t\rho_\lambda)^2)(\mu_\lambda''\\&+t\rho_\lambda\mu_\lambda''+t\mu_\lambda\rho_\lambda'')]/[2(\mu_\lambda^2+(1-t\rho_\lambda)^2)^{\frac{5}{2}}],\\(K_\delta)_t &= \frac{4\mu_\lambda^2\rho_\lambda^3(t\rho_\lambda-1)}{(\mu_\lambda^2+(1-t\rho_\lambda)^2)^3},\\(K_\delta)_s &= [2\mu_\lambda\rho_\lambda(\mu_\lambda^2\mu_\lambda'\rho_\lambda+t\rho_\lambda^2(t\mu_\lambda\rho_\lambda'\\&+2\mu_\lambda')-\mu_\lambda'\rho_\lambda-t^2\rho_\lambda^3\mu_\lambda'-\mu_\lambda^3\rho_\lambda'\\&-\mu_\lambda\rho_\lambda')]/[(\mu_\lambda^2+(1-t\rho_\lambda)^2)^3].\end{aligned}$$

From equation (6), the following result is obtained

$$\begin{aligned}&3\mu_\lambda\rho_\lambda^2(t\rho_\lambda-1)[(\mu_\lambda'(t\rho_\lambda-1)-t\mu_\lambda\rho_\lambda')(\mu_\lambda\mu_\lambda'-t\rho_\lambda'\\&+t^2\rho_\lambda\rho_\lambda')+(\mu_\lambda^2+(1-t\rho_\lambda)^2)(\mu_\lambda''+t\mu_\lambda\rho_\lambda''+t\rho_\lambda\mu_\lambda'')]\\&=(\mu_\lambda^2+(1-t\rho_\lambda)^2)^4[(\mu_\lambda\rho_\lambda'-\mu_\lambda'\rho_\lambda)(\mu_\lambda^2+(1-t\rho_\lambda)^2)\\&-3(\rho_\lambda-t\rho_\lambda^2)(\mu_\lambda'(1-t\rho_\lambda)-t\mu_\lambda\rho_\lambda')](\mu_\lambda^2\mu_\lambda'\rho_\lambda-\mu_\lambda'\rho_\lambda\\&-\mu_\lambda^3\rho_\lambda'-\mu_\lambda\rho_\lambda'-t^2\rho_\lambda^3\mu_\lambda'+t^2\rho_\lambda^2\mu_\lambda\rho_\lambda'+2t\rho_\lambda^2\mu_\lambda')).\end{aligned}$$

Example. Let a unit speed curve λ be given as

$$\lambda(s) = \left(\frac{1}{\sqrt{2}}cos(s), \frac{1}{\sqrt{2}}sin(s), \frac{s}{\sqrt{2}}\right).$$

Then, Serret-Frenet frame and curvatures of γ are obtained as

$$\boldsymbol{T}_\lambda = \lambda' = (-\frac{1}{\sqrt{2}}sin(s), \frac{1}{\sqrt{2}}cos(s), \frac{1}{\sqrt{2}}),$$
$$\boldsymbol{N}_\lambda = \frac{\lambda''}{\|\lambda''\|} = (-cos(s), -sin(s), 0),$$
$$\boldsymbol{B}_\lambda = \boldsymbol{T}_\lambda \times \boldsymbol{N}_\lambda = (\frac{1}{\sqrt{2}}sin(s), \frac{-1}{\sqrt{2}}cos(s), \frac{1}{\sqrt{2}}),$$

and

$$\mu_\lambda = <\boldsymbol{T}_\lambda', \boldsymbol{N}_\lambda> = \frac{1}{\sqrt{2}},$$
$$\rho_\lambda = <\boldsymbol{N}_\lambda', \boldsymbol{B}_\lambda> = \frac{1}{\sqrt{2}}.$$

Let δ be adjoint curve of λ. Then, we have

$$\delta(s) = \int \boldsymbol{B}_\lambda(s)ds$$
$$\delta(s) = (-\frac{1}{\sqrt{2}}cos(s), \frac{-1}{\sqrt{2}}sin(s), \frac{s}{\sqrt{2}}) + c,$$
$$\boldsymbol{T}_\delta = (\frac{1}{\sqrt{2}}sin(s), \frac{-1}{\sqrt{2}}cos(s), \frac{1}{\sqrt{2}}),$$
$$\boldsymbol{N}_\delta = (cos(s), sin(s), 0),$$
$$\boldsymbol{B}_\delta = (-\frac{1}{\sqrt{2}}sin(s), \frac{1}{\sqrt{2}}cos(s), \frac{1}{\sqrt{2}}),$$
$$\mu_\delta = \frac{1}{\sqrt{2}}, \quad \rho_\delta = \frac{1}{\sqrt{2}}.$$

Also, the normal surface of δ is expressed as

$$\varphi^\delta(s,t) = \delta + t\boldsymbol{N}_\delta.$$

Then, we obtain

$$\varphi^\delta_s = (1 - \frac{t}{\sqrt{2}})\boldsymbol{T}_\delta + \frac{t}{\sqrt{2}}\boldsymbol{B}_\delta,$$
$$\varphi^\delta_{ss} = (\frac{1}{\sqrt{2}} - t)\boldsymbol{N}_\delta,\ \varphi^\delta_{st} = \frac{\boldsymbol{B}_\delta - \boldsymbol{T}_\delta}{\sqrt{2}},$$
$$\varphi^\delta_t = \boldsymbol{N}_\delta, \quad \varphi^\delta_{tt} = 0.$$

and, from the equalities, it's obtained

$$n_\delta = \frac{(\sqrt{2}-t)\boldsymbol{T}_\delta + t\boldsymbol{B}_\delta}{\sqrt{2+2t^2-2\sqrt{2}t}}.$$

Then, we obtain

$$E = t^2 - \sqrt{2}t + 1,\ F = 0, \quad G = 1,$$
$$e = 0, \quad f = \sqrt{\frac{1}{2t^2-2\sqrt{2}t+2}}, \quad g = 0.$$

Therefore, the first and the second fundamental forms, the mean and the Gaussian curvature and the principal curvatures of the normal surface are obtained as

$$\boldsymbol{I}_\delta \quad = (t^2 - \sqrt{2}t + 1)ds^2 + dt^2,$$
$$\boldsymbol{II}_\delta \quad = \sqrt{\frac{2}{t^2 - \sqrt{2}t + 1}}dsdt,$$
$$K_\delta = \frac{-1}{2(t^2 - \sqrt{2}t + 1)^2}, \qquad H_\delta = 0,$$
$$k_{\delta_1} = \frac{1}{(\sqrt{2}t^2 - 2t + \sqrt{2})}, \quad k_{\delta_2} = \frac{-1}{(\sqrt{2}t^2 - 2t + \sqrt{2})}.$$

Hence, this normal surface example, where we try to exemplify the normal surface of the adjoint curve of a regular curve with constant curvature and torsion, provides the claims of Theorem 7, Corollary 11, Theorem 12 and Corollary 13-(i).

5. Conclusion

Our general aim in this study was to present a specific study on curves and surfaces, which are important topics in differential geometry. We have given the characterizations of the normal surfaces of the adjoint curves we have studied in 3D Euclidean space by obtaining the time-dependent equations of motion. Here, we obtained mean curvature, which gives results about the minimality of the surface, and Gaussian curvature and principal curvatures, which give results about the geometric shapes of the surface under certain conditions. In addition, by examining the cases where such surfaces are minimal and flat, we revealed the relationship between these two cases.

We hope that this study, under the general title of curves and differential geometry of surfaces, will make a specific contribution to studies in the fields of mathematics and applied sciences. Our next study will be about some special surfaces of adjoint curves.

Contributions of the authors

The authors contributed equally to the article.

Conflict of Interest Statement

There is no conflict of interest between the authors.

Statement of Research and Publication Ethics

The study is complied with research and publication ethics.

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