Journal of Theoretical Educational Science, 14(4), 638-664, October 2021
Kuramsal Eğitimbilim Dergisi, 14(4), 638-664, Ekim 2021
[Online]: http://dergipark.org.tr/akukeg
DOI number: http://doi.org/10.30831/akukeg.888232

Pre-Service Teachers' Understandings of Doing Statistics in the Context of Teaching Graphs*

Öğretmen Adaylarının Grafiklerin Öğretimi Bağlamında İstatistik Yapmaya İlişkin Anlayışları

Nadide YILMAZ** **İ. Elif YETKİN-ÖZDEMİR*****

Received: 28 February 2021 **Research Article** **Accepted:** 03 September 2021

ABSTRACT: It was aimed to examine pre-service teachers' developing understandings of doing statistics within a lesson study. It was focused on their understandings (i.e., content knowledge) of doing statistics as well as how they transform their understandings (i.e., content knowledge) into teaching practices (i.e., knowledge of student & knowledge of teaching) while designing and implementing lessons related to graphs. Three senior pre-service teachers participated in a two phase (university and school classroom) lesson study. Data were collected through lesson plans prepared by the pre-service teachers in groups, observations, field notes, semi-structured interviews and video and audio recordings of the group meetings and instructional implementations. Results showed that they did not consider doing statistics as it consisted of many inter-related components initially. Their understandings developed greatly as they designed lesson plans, discussed the concepts and reflected on their lesson plans and instructional implementations. Their starting to put statistical questions into the center of doing statistics became an important turning point that affected their conceptions related to other themes (e.g., collecting data, interpreting graphs). It was also observed that their understandings related to doing statistics impacted their teaching practices.

Keywords: Doing statistics, teaching graphs, lesson study, pre-service teachers, middle school grades.

ÖZ: Bu çalışmada öğretmen adaylarının istatistik yapma konusundaki anlayışlarının ders imecesi/araştırması bağlamında incelenmesi amaçlanmıştır. Öğretmen adaylarının istatistik yapma sürecindeki anlayışlarının (alan bilgisi) grafiklerle ilişkili ders planlarken ve uygularken nasıl değiştiği ve nasıl öğretim pratiklerine dönüştüğüne (öğrenci ve öğretim bilgisi) odaklanılmıştır. Çalışmaya üç son sınıf öğretmen adayı katılmış, iki aşamadan oluşan (üniversite ve gerçek okul) ders imecesi/araştırması uygulaması gerçekleştirmişlerdir. Öğretmen adayları tarafından grup halinde hazırlanan ders planları, gözlemler, alan notları, yarı yapılandırılmış görüşmeler ve grup toplantılarının video ve ses kayıtları ve öğretim uygulamaları aracılığıyla veriler toplanmıştır. Öğretmen adayları başlangıçta istatistik yapma sürecinin birbiriyle ilişkili birçok bileşenden oluştuğunu düşünmemişlerdir. Ders planları tasarladıkça, kavramlar hakkında tartıştıkça anlayışları gelişmiş; bu gelişim ders planları ve öğretimsel uygulamalarına büyük ölçüde yansımıştır. İstatistiksel soruları istatistik yapmanın merkezine koymaya başlamaları, diğer temalarla ilgili fikirlerini (örneğin, veri toplama, grafikleri yorumlama) etkileyen önemli bir dönüm noktası haline gelmiştir. Ayrıca istatistik yapmakla ilgili anlayışlarının öğretim uygulamalarını doğrudan etkilediği de gözlemlenmiştir.

Anahtar kelimeler: İstatistik yapma, grafiklerin öğretimi, ders imecesi/araştırması, öğretmen adayları, ortaokul seviyesi.

* This study is derived from PhD dissertation of first author conducted under the supervision of second author. This research was financially supported by the Scientific and Technological Research Council of Turkey (TÜBİTAK) under grant 2211-A.
** *Corresponding Author*: Asst. Prof. Dr., Karamanoğlu Mehmetbey University, Karaman, Turkey, nadideylmz70@gmail.com, https://orcid.org/0000-0003-1624-5902
*** Assoc. Prof. Dr., Hacettepe University, Ankara, Turkey, elif.yetkin.ozdemir@gmail.com, https://orcid.org/0000-0001-8784-0317

Citation Information
Yılmaz, N., & Yetkin-Özdemir, İ. E. (2021). Pre-service teachers' understandings of doing statistics in the context of teaching graphs. *Kuramsal Eğitimbilim Dergisi [Journal of Theoretical Educational Science], 14*(4), 638-664.

ISSN: 1308-1659

The increasing need for statistics has led to increased awareness of the importance of this field (Eicher & Zapata-Cardana, 2016). This awareness has also been reflected in education and "how the teaching and learning of statistics should be" has become one of the important research topics (Batanero et al., 2011; Shaughnessy, 2007). Even though there are similarities between statistics and mathematics, some fundamental differences gave rise to the need to address the learning and teaching of these fields separately (Garfield & Ben-Zvi, 2004; Groth, 2007). Although mathematics is used in statistics, non-mathematical issues (e.g., deriving meaning from data by using context) also play an important role in doing statistics (delMas, 2004; Groth, 2007; Rossman et al., 2006). This gave rise to the recognition that the statistics needed for teaching contain different information than teaching mathematics. Researchers have pointed out that this knowledge should be defined (Groth, 2007; Moore, 1988).

Four components of doing statistics are emphasized for learning and teaching statistics: (1) formulating questions, (2) data collection, (3) analyzing data and (4) interpreting results (Bargagliotti et al., 2020; Carver et al., 2016; Franklin et al., 2005; Ministry of National Education [MoNE], 2018; National Council of Mathematics [NCTM], 2000). Formulating questions refers to identifying the problem situation and defining the question(s) that can be answered by data (Franklin et al., 2005). At the heart of a statistical research, there are questions that can be answered by collecting and analyzing data (Arnold, 2008). The questions should allow data collection (English et al., 2017; NCTM, 2000), contain a specific purpose (Graham, 2006) and related to a context. Also, the features of the group from which data will be collected should be clearly explained when formulating questions (English et al., 2017). Data collection is another component of doing statistics. This involves making and implementing plans to gather appropriate data (Schwartz, 2008). Selecting the most suitable and efficient data collection methods and collecting data appropriately are important skills in statistics. On the basis of the questions, decisions should be made about the variables (e.g., achievement, preference, etc.), data collection tools (e.g., observation, interview, questionnaire) and the population and sampling methods (Bargagliotti et al., 2020; Franklin et al., 2005). In addition, issues such as reaching the target population and ensuring the independence of observations are important points to be taken into consideration (Bargagliotti et al., 2020). Analysis of the data, another component of doing statistics, involves selecting and applying suitable data analysis methods (Bargagliotti et al., 2020; Franklin et al., 2005). Organizing data in terms of tables and graphs, calculating measures of central tendency and measures of dispersion are necessary in order to answer the research questions (MacGillivray & Pereira-Mendoza, 2011). The last component of doing statistics is interpreting results. At this component, meanings are derived from the findings and they are associated with the research questions asked at the beginning (Franklin et al., 2005). The interpretation process is made up of two levels (Curcio, 1987; Friel et al., 2001). The first level is reading between the data, where the reader tries to make sense of the data by performing quantitative comparisons (larger, less) and mathematical operations (addition, subtraction, multiplication, division). The next level involves reading beyond the data. At this level, the information in the graph and the reader's previous knowledge are integrated, and from here, expansions, inferences, and predictions are made (Curcio, 1987; Friel et al., 2001).

All components of doing statistics has been emphasized and specific expectations related to doing statistics as a research process were stated on the current middle school mathematics curriculum (MoNE, 2009, 2013, 2018). However, researchers argue that the components of doing statistics are usually covered separately in instructional practices, which could constraint students to develop statistical understandings (English, 2014; Güven et al., 2015; Hacısalihoğlu-Karadeniz, 2016; Öz, 2019; Pfannkuch, 2005). Numerous studies have reported that both teachers and pre-service teachers consider concepts and ideas related to statistics as separated (Batanero et al., 2010; Burgess, 2001, 2002; Chick & Pierce, 2008; Gürel, 2016; Heaton & Mickelson, 2002; Ijeh, 2012; Kurt, 2015; Leavy, 2006; Mercimek, 2013; Reston et al., 2006; Santos & Ponte, 2014; Sorto, 2004), which might influence their teaching of statistics. For instance, Sorto (2004) revealed that pre-service teachers had difficulty associating the questions they formulated with graphical representations. Likewise, Batanero et al. (2010) observed that only one-third of the pre-service teachers who participated in their study were able to associate their comments with the formulated question while interpreting the graphs. Several studies have also revealed that both pre-service and in-service teachers focused on procedural aspects such as drawing graphs or making calculations correctly rather than emphasizing and relating the components of doing statistics (Chick & Pierce, 2008; Heaton & Mickelson, 2002; Ijeh, 2012; Mercimek, 2013; Reston et al., 2006).

Students need more opportunities to formulate questions, collect data and interpret results based on the formulated questions to develop statistical thinking skills (Ader, 2018; Makar & Fielding-Wells, 2011). This requires that pre- and in-service teachers have the knowledge and skills related to the components of doing statistics (Heaton & Mickelson, 2002; Makar & Fielding-Wells, 2011). Teachers' having the necessary knowledge and skills to develop these understandings undoubtedly plays an important role in the success of the targeted statistics education (Carver et al., 2016; Franklin et al., 2015; Van de Walle et al., 2010). For teacher educators, how pre-service teachers will acquire this knowledge and skills is an important problem because the teaching-learning process has a more complex structure than it is thought (Grossman et al., 2009; Hiebert et al., 2007; Morris et al., 2009). At this point, it is very important to develop learning environments that allow pre-service teachers to practice as well as to acquire theoretical knowledge (Zhang & Cheng, 2011). Although experiences in university classrooms offer valuable insights, they are usually limited in developing knowledge and skills provided by real classroom environments (Cohan & Honigsfeld, 2006; Grossman et al., 2009). The lesson study model allows pre-service teachers to work collaboratively in planning, implementing and evaluating lessons toward a common goal. In this way, the model facilitates teacher knowledge and skills (Borko, 2004; Fernandez, 2005; Meyer & Wilkerson, 2011; Murata, 2010; Wright, 2009; Yamnitzky, 2010; Zhang & Cheng, 2011).

This study aimed to examine pre-service teachers' developing understanding of doing statistics within a lesson study that lasted for three months. We focused on their understandings (i.e., content knowledge) of doing statistics as well as how they transform their understandings (i.e., content knowledge) into teaching practices (i.e., knowledge of content and student & knowledge of content and teaching) while designing and implementing lessons related to graphs. The study's findings could inform researchers about teacher learning of doing statistics and how their understandings could evolve with practice in the university and real-classroom environments. In this way, findings could also reveal ideas for teacher educators in structuring pre-service teacher education programs.

Method

Holistic single-case study was utilized. The case examined in this study is the group of three pre-service teachers who conducted three lesson study cycle in their last year of the undergraduate program training middle school mathematics teachers. Criterion sampling was used when selecting the participants. Among 12 pre-service teachers who completed the required content and pedagogical content courses (e.g., statistics and probability, methods of teaching mathematics) and volunteered to participate in the study, Gamze, Şirin and Beyza[1] were selected for in-depth analysis. The interviews and small group work at the beginning of the study showed that this group took more active roles and expressed themselves better than other volunteers. They also demonstrated typical difficulties and conceptions reported in the literature (e.g. focusing on the procedural aspects, ignoring the process of doing statistics). The fact that the great majority of the students attending teacher education programs are females resulted in all the participants of the current study being females. The group focused on seventh grade learning objectives related to graphs: (1) Constructs a pie graph of a data set and interprets it (2) Constructs a line graph of the data and interprets it and (3) Shows the data related to the research questions with a suitable representation (pie graph, a frequency table, a bar graph or a line graph) and makes conversions between the representations. The pre-service teachers were expected to design, conduct and revise three lesson plans during the spring semester of the 2016-2017 academic year. They were asked to prepare lesson plans according to a format with four components: learning activities, expected student responses, teacher's responses, goals and methods of evaluation (Figure 1).

Figure 1

Sample Lesson Plan

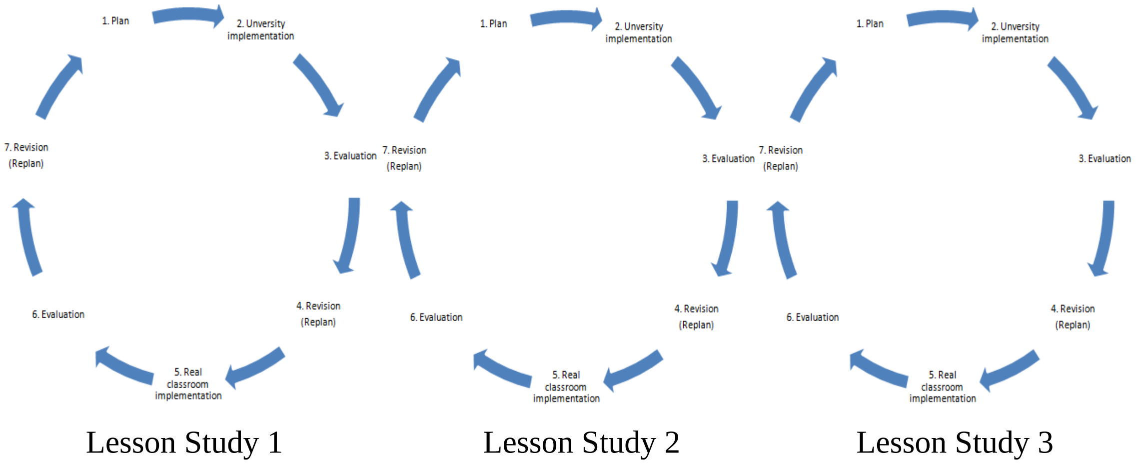
Learning activities	Expected student responses	Teachers'responses	Goals and methods of evaluation
Giriş – Çocuklar, şu ana kadar sütun ve daire grafiğini öğrendik. Peki bir düşünelim bakalım; bir ilin 5 ay boyunca ortalama sıcaklıklarını bilsek nasıl bir grafik kullanırız?	- Sütun grafiği hocam. - Daire grafiği hocam.	– Bu durum için, bu grafik türlerinden daha uygun bir grafik türü öğreneceğiz.	Günlük hayatla bağdaştırmak, öğrencileri motive etmek.

[1] All names are pseudonymous

Then each group member implemented one lesson plan primarily in the university classroom and then in the real middle school classroom. The lesson plans were conducted by Gamze, Şirin and Beyza, respectively. After each implementation, the lesson was evaluated and revised by the group members. For the university implementation, both researchers (the first and the second author) and other pre-service teachers in the program participated in and evaluated the implementation of the lesson. For the real middle school classroom implementation, the first author and the mentor teacher were present and evaluated the implementation with the group members. This procedure can be seen in Figure 2.

Figure 2

Lesson Study Model (Adapted from Zhang & Cheng, 2011)



Instructional activities in the university and real classroom environment as well as the meetings for planning, evaluating and revising the lesson plans, were video-recorded and transcribed into documents. Lesson plans prepared by the pre-service teachers, video-recorded and transcribed lesson study meetings (for planning, implementation and evaluation) and classroom instructions (university implementation and real classroom implementation), semi-structured interviews, observation forms filled out by the observers, field notes taken by the first author and reflective papers written by the pre-service teachers were used as data collection tools. Data collected by these tools are used to describe pre-service teachers' decisions and actions reflecting their initial and evolving understandings at each phase of three lesson study cycle.

The descriptive analysis method was employed. Each cycle of lesson study depicted in Figure 2 (plan, university implementation, evaluation, revision/replan, real classroom implementation, evaluation, revision/replan) was analyzed separately. We analyzed the group's all instructional decisions (implemented or ignored) and actions related to teaching statistics. For instance, we examined the tasks and activities they designed to introduce the topic, the questions they asked the students to explore the topic as well as their explanations and responses to students' questions. We first coded these instructional decisions and actions to describe how the pre-service teachers address each component of doing statistics (i.e., formulating statistical questions, collecting, analyzing and interpreting data) when designing and implementing lessons. Interpreting these codes within and across each component of doing statistics (i.e.

formulating statistical questions, collecting, analyzing and interpreting data) revealed some themes as important understandings for teaching statistics. For instance, understanding the role of context emerged as a new theme related to each component of doing statistics. Likewise, understandings related to variable type emerged as another theme. We observed that pre-service teachers struggled with designing and implementing lesson plans when they did not take into account the role of context and variable type and decided to include these themes as necessary understandings for teaching statistics.

Furthermore, understanding the roles of graphs emerged as another theme since the pre-service teachers were aimed to teach graphs in the lesson study. Therefore, seven themes emerged: understandings related to the (1) statistical questions, (2) data collection, (3) the role of graphs, (4) the role of context, (5) variable type, (6) graph construction and the elements of the graph and (7) reading and interpreting graphs. Under each theme, we also explored whether and to what extend pre-service teachers transform their understanding into teaching practices. Finally, analyses from each cycle of lesson study were compared to seek any changes and development in pre-service teachers' understandings of doing statistics and teaching practices.

Various measures were taken to ensure the validity and reliability of the data. Purposive selection of the participants and the use of different data collection tools are thought to have strengthened the transferability of the study and contributed to credibility and dependability. In addition, presentation of the data collection and data analysis processes to the reader through detailed explanations can be argued to have contributed to not only the transferability and dependability of the study but also to its confirmability. The code list was created by the authors together. The first author coded the raw data (documents and observations) at different periods (at an interval of three months). The rate of agreement between these two coding was found to be 90%. In addition, an expert coded twenty-five percent of the data, and the inter-coder reliability was found to be 85%. On the points where there was disagreement in the coding, the authors and the expert were discussed together and reached a consensus. Thus, it can be argued that the current study satisfies the criterion of validity (Miles & Huberman, 1994). On the basis of these results, it can be said that the current study is credible and dependable.

Ethical Procedures

Ethical approval was sought from Provincial Directorate for National Education, the participating school and university for video-recordings of the teachers to be used in the study. Also, the name of prospective teachers remained anonymous. The ethical committee approval date is May 2, 2016 and the number of their approval document is 35853172/433-1358.

Results

Findings of the study are presented under seven themes emerged: understandings related to the (1) statistical questions, (2) data collection, (3) the role of graphs, (4) the role of context, (5) variable type, (6) graph construction and the elements of the graph and (7) reading and interpreting graphs. Under each section, we described the groups' understandings and how they transformed their understanding into instructional practices. In order to depict the development observed over the phases of three lesson study cycles, findings are summarized in tables with sample criteria related to each theme.

Understandings Related to the Statistical Questions

The development of pre-service teachers' understandings and their instructional practices related to statistical questions are summarized in Table 1.

Table 1

Understandings Related to the Statistical Questions

	1st Lesson Study		2nd Lesson Study		3rd Lesson Study	
Criteria	UC	RC	UC	RC	UC	RC
Thinking and discussing about statistical questions	-	-	-	√	-	√
Supporting students to formulate their own statistical questions	-	-	-	√	√	√
Including a problem statement or a purpose in the tasks	-	-	-	√	-	√
Supporting students to establish the relationship between the questions formulated in the task and the type of graph	-	-	-	√	√	√

UC: University classroom environment

RC: Real middle school classroom environment

In the lesson study related to the first objective (constructing and interpreting pie graph)[1], it was observed that the pre-service teachers did not take into consideration the fact that doing statistics starts with statistical questions. They presented a bar graph and asked students to construct the same data with a pie graph without any reason or purpose. Their statements such as "*let's convert the bar graph into a pie graph"* (*University Planning-Lesson study 1)* indicate their lack of attention for the purpose of drawing graphs. Even after the expert (the second author) emphasized the need for a purpose by stating, *"Why do we construct a pie graph? We need to think about it."* the pre-service teachers did not make any revisions on their plans. Likewise, they didn't consider statistical questions as they planned for the second objective (constructing and interpreting line graphs)[2]. Şirin made an introduction to the lesson by saying, *"Let's suppose I present you 5-day average temperatures of a city. I want to express this data in a graph. What kind of graph should I use?" (University Implementation-Lesson study 2)*. The expert, again, brought up the fact that there should be a purpose: *"For what purpose do we use statistics? Why do we compare here [temperatures]?" (University Evaluation-Lesson study 2)*. After the evaluation meeting, pre-service teachers revised the task as follows:

[1]Constructs a pie graph of a data set and interpret it (7th grade).
[2]Constructs a line graph of the data and interpret it (7thgrade).

Figure 2

Revised Task-Lesson Study 2

"Uncle Hasan will plant a vegetable in his garden. In order to get the highest yield from this vegetable, the temperature change should be the least for 2 days after the day it is planted. Below is the weather forecast for the next 14 days. Let's find the best days for Uncle Hasan to plant."

1	2	3	4	5	6	7	8	9	10	11	12	13	14
20°C	23°C	25°C	22°C	19°C	23°C	26°C	24°C	25°C	26°C	25°C	23°C	20°C	24°C

Here, the question pre-service teachers formulated is structured around a problem statement and includes an objective. During the implementation in the real classroom environment, Şirin asked questions to test whether a bar graph would be suitable for answering the question as shown below:

> Real Classroom Implementation-Lesson study 2
>
> Şirin: Now, which graphs have we learned?
>
> Student: Bar graph, pie graph, tally, frequency table.
>
> Şirin: Yes, we have learned these graphs. So, what kind of graph was the bar graph? Let's construct a bar graph. Let's see whether it is a suitable graph for these data or for the thing I am exploring [i.e., which days are the best days to plant]?

By asking questions, Şirin encouraged students to consider the problem as well as the relationship between the problem and the type of graph to be used. Even though the pre-service teachers began to understand the role of statistical questions, developing a deeper understanding seems to take more time and experience. The task they designed for the third objective (selecting and converting representations)[1] did not include a problem situation and purpose at first: *"The table presents favourite school subjects in the class 7/A. Construct the appropriate graph according to the table. Select three school subjects, interpret their status and find the percentages."* After the implementation in the university classroom, the lack of a statistical question (a problem or a purpose) in the task came up as an issue again. The group revised the task to include a problem statement and a purpose: *"The table presents favourite school subjects in the class 7/A. The teacher wants to know whether mathematics is a popular subject in the school. Construct the appropriate graph according to the table. Interpret the status of mathematics and two school subjects you selected."* The revised task was formulated around a question: *"Is math a favourable school subject among students?"* This revision shows that even though pre-service teachers still struggle with thinking and discussing about statistical questions, they began to design instructional tasks including a problem statement or a purpose for collecting or analyzing data with some help.

Understandings Related to the Data Collection

The development of pre-service teachers' understandings and their instructional practices related to data collection are summarized in Table 2.

[1]Shows the data related to the research questions with a suitable representation (pie graph, a frequency table, a bar graph or a line graph) and makes conversions between the representations (7th grade).

Table 2

Understandings Related to the Data Collection

	1st Lesson Study		2nd Lesson Study		3rd Lesson Study	
Criteria	UC	RC	UC	RC	UC	RC
Relating data collection with statistical questions		-	-	-	-	-
Taking into account decisions about data collection (e.g., data collection tools, selecting participants)	-	-	-	-	-	-
Allowing students to think about collected data	-	-	√	√	√	√
Simulate simple data collection with students	√	-	-	-	-	-

UC: University classroom environment,

RC: Real middle school classroom environment

With regard to the first lesson plan related to constructing and interpreting pie graphs, the group included an activity to collect data. In the university classroom implementation, Gamze asked students about their favorite football team and made a list on the board. Even though she attempted to show students the process of data collection, she neither made explicit connections with the formulated question or purpose nor the selection of the participants. After this issue was discussed in the evaluation meeting, pre-service teachers revised their lesson plan for the real classroom implementation. Rather than collecting data in the classroom, Gamze showed students the data set and asked them how it could have been collected. However, she did not let students think about the data collection process and immediately responded to her own question: *"Now, do you know how we obtained these data? [without waiting for student response] You know we are enrolled in a university. I asked my classmates. The data belong to them…"* Here, students did not have opportunities to explore and discuss important elements of the data collection process (e.g., data collection tools, selecting the participants). After the evaluations on the first study lesson, pre-service teachers began taking more attention to let students think about data collection. During the real classroom implementation, Şirin asked the students how the data might have been collected and waited for the students' responses.

> Real Class Implementation-Lesson study 2
>
> Şirin: Now you can see the 14-day temperature change [pointing to the data on the board.] How do you think we have obtained the data? Let's first talk about it.
>
> Student: You may have obtained from the weather forecast.

Even though these instances show some progress in their understanding of data collection, they failed to explore detailed analysis of data collection and to relate the data collection process with the question formulated in all three lesson study cycles.

Understandings Related to the Role of Graphs

The development of pre-service teachers' understandings and their instructional practices related to the role of graphs are summarized in Table 3.

Table 3

Understandings Related to the Role of Graphs

	1st Lesson Study		2nd Lesson Study		3rd Lesson Study	
Criteria	UC	RC	UC	RC	UC	RC
Taking into account the role of graphs as organizing and representing data	√	√	√	√	√	√
Taking into account the role of graphs in answering statistical questions	-	√	-	√	√	√
Designing tasks to explore how different types of graphs display data differently	-	-	-	√	√	√
Supporting students to compare different types of graphs in relation to the formulated statistical questions	-	√	-	-	√	√

UC: University classroom environment

RC: Real middle school classroom environment

The pre-service teachers were observed to be focusing on the roles attributed to the graphs as tools for organizing and representing data. They ignored graphs' role as tools for answering statistical questions. For instance, when one student asked why they need to learn about line graphs in addition to pie graphs, Gamze explained that each type of graph provides a different way of representing data. She, however, did not focus on why it is important to represent data in different ways. This case shows that pre-service teachers have not understood that different types of graphs allow us to represent data in different ways, which helps us answer different statistical questions. This issue was discussed after the implementation. The pre-service teachers revised their lesson plan to explain that line graphs make it possible to compare changes in data sets. The revised plan also emphasized the idea that pie graphs allow us to see the ratio of parts to the whole data set. These explanations show that the pre-service teachers started to realize that pie and line graphs are tools for representation and tools for answering statistical questions. In the real classroom implementation, Gamze provided the opportunity for students to think about the function of the pie graph. When one student based her reasoning for using pie graphs on angles, Gamze emphasized that pie graphs make it easy to see the relationship between the part and the whole.

For the second objective, the pre-service teachers wanted to focus on the fact that line graph allows seeing the change in a data set. They decided to show students a line graph representing temperatures of a city and ask questions about the changes of temperatures. Şirin's statements such as "*Why do you think we connect them [points of change]?*" and *"You can see the decrease [in temperature] more easily in this way,* *can't you? For example, from Monday to Tuesday, it [the temperature] decreased"* illustrates her effort to make the role of line graph as a tool to show change more explicit for the students.

Despite these instances, it was observed that the pre-service teachers still struggled with their understandings of graphs as tools to answer statistical questions into their teaching practices. For instance, one student suggested using a bar graph to represent the data (i.e., temperatures of a city). Her reasoning was that they could also see the change with bars. Here, Şirin merely confirmed her suggestion rather than using this opportunity to associate the role of graphs with the statistical question asked at the beginning. Here, the student's suggestion could have been used to explore the elements of line and bar graphs in relation to answering particular statistical questions: "lines are connecting the points to examine changes," whereas "height of bars help us to compare frequencies of categories." After implementing the lesson, the group discussed the statistical questions that can be answered by line graph. In the revised plan, they decided to ask the students to compare the problems that could be answered with a bar graph with the problems that could be answered with a line graph.

In the lesson planning meetings and implementations about the third objective, it was observed that the pre-service teachers focused more on the fact that graphs are tools to respond to statistical questions. During the university implementation, when Beyza noticed that students focused on the keywords while determining the suitable type of graph, she guided the students to think about the problem statement. The related section of the lesson is presented below.

> University Implementation-Lesson study 3
>
> Student: For example, when there is a temperature change or population change, I can use the line [graph].
>
> Beyza: Hımm. Only for temperature or population? What is important here, the temperature or the change [of the temperature]?
>
> Student: The change.
>
> Beyza: Yes, when you want to show something is changing [you'll use line graph].

Here, Beyza asked questions to understand if students focused on keywords when selecting the type of graph. Then she emphasized the necessity of making connections with a problem statement or a purpose. In the real classroom implementation, Beyza noticed that a group of students had difficulty understanding the roles of bar and line graphs. She asked questions such as "*When I construct a bar graph, what can I see? What can we see in the line graph?"* to help students focus on the distinctive characteristics each type of graph has. These findings show the development of groups' understandings about the role assumed by graphs in answering the formulated question.

Understandings Related to the Role of Context in Doing Statistics

The development of pre-service teachers' understandings and their instructional practices related to the role of context in doing statistics are summarized in Table 4.

Table 4

Understandings Related to the Role of Context in Doing Statistics

	1st Lesson Study		2nd Lesson Study		3rd Lesson Study	
Criteria	UC	RC	UC	RC	UC	RC
Designing tasks involving meaningful contexts leading to statistical investigation	-	-	-	√	√	√
Considering students' overgeneralizations related to common contexts in statistical investigations	-	-	-	√	√	√
Designing tasks to reveal and overcome students' overgeneralizations related to common contexts in statistical investigations	-	-	-	√	√	√

UC: University classroom environment

RC: Real middle school classroom environment

While the pre-service teachers were planning their lessons in relation to the first objective, they considered the context highly restricted. For example, they included the following questions in their lesson plan *"Who supports which football team? If we organized and showed your responses on a graph, which graph would you use?" (University Plan-Lesson study 1)*. Even though the task constitutes a context related to daily life, it does not include a meaningful purpose: why do we need to know who supports which football team? Who wants to know this data? This approach caused them to experience difficulty when one student asked whether they could use a pie graph for representing a data set involving temperatures of a city. The following excerpt shows Gamze's response to the student:

University Implementation-Lesson study 1

Student: How about using a pie graph for displaying temperature?

Gamze: You mean using a pie graph for displaying temperature? Tell me, what degrees of temperature [asking for the data]?

Student: Let's say -2 and 5 degrees [in Celsius].

Gamze: Hımm, are these for two different days? Do you think that representing these values [in pie graph] would be meaningful? Here [referring to the pie graph], we can see the number of people [referring to frequencies], can't we? For instance, the largest sector shows the highest number of people [frequency]. Here, how would I show 25 degrees [of temperature] [on pie graph]?

….

Student: That means we cannot use the pie graph for every case, can we?

Gamze: Yes. Each graph is suitable for certain cases.

Here Gamze's conception of context is limited to a case or a situation (e.g., temperature of a city, favourite sport teams) and data. Hence, she could not produce a new problem statement related to the given context that may require displaying data with pie graph.

While thinking about a context for the second objective, pre-service teachers paid great attention to the variable type and their being realistic. Yet, they still ignored the problem statement and purpose. Like the first study lesson, they presented a data set with a real-life situation without any problem statement or purpose (i.e., constructing a line graph for degrees of temperature of a city). After the evaluation of the delivered lesson, they revised the context around a certain goal (i.e., finding the best days for Uncle Hasan to plant his vegetables). During the implementation, Şirin asked students in which situations the use of a line graph would be suitable and evaluated students' responses by emphasizing the use of line graphs as displaying the change.

While determining contexts related to the third objective, the pre-service teachers tried to avoid typical contexts and attended to students' common misconceptions. They stated that *"Everybody thinks that votes should be represented on a pie graph. We selected this context on purpose; we wanted them to learn that when necessary, votes can be shown with a bar graph." (University Evaluation-Lesson study 3)*. The task is presented as follows.

Figure 3

University Implementation-Bar Graph-Lesson Study 3

"…middle school vote counts for school president

Candidates	*Vote count*
Pelin	*78*
Veli	*57*
Alparslan	*99*
Aysel	*127*

In the table, the number of votes for the school president candidates in the … middle school is given. According to these data, draw a graph where you can compare the data and determine the president and vice president by looking at the graph."

Though the task includes a problem statement, the expression "compare the data" hints for students to construct a bar graph. Yet, the pre-service teachers' addressing an overgeneralization (i.e., the number of votes are represented with a pie graph) can be seen as a development from an instructional point of view.

During the university implementation, Beyza wanted students to present sample contexts for using different types of graphs. She talked about the importance of purpose when deciding the most suitable type of representation and stressed that the type of the variable is not the only criterion to be taken into account. During the implementation, the pre-service teachers observed that students tend to match certain contexts with certain types of graphs. In order to prevent these generalizations, they allowed students to discuss the context and the problem situation related to the context. While a line graph can be used when the change in the population is questioned, a bar graph is more suitable to represent the distribution of population across the years.

Understandings Related to the Variable Type

The development of pre-service teachers' understandings and their instructional practices related to the variable type are summarized in Table 5.

Table 5

Understandings Related to the Variable Type

	1st Lesson Study		2nd Lesson Study		3rd Lesson Study	
Criteria	UC	RC	UC	RC	UC	RC
Considering the type of the variable as an important criterion in data analysis (i.e., in selecting the appropriate graphic representation)	√	√	√	√	√	√
Evaluating variable type within the context of the formulated question	-	-	-	√	√	√
Considering students' overgeneralizations regarding the relationship between the type of variable and the type of graph	-	-	-	√	√	√
Designing tasks to reveal and overcome students' overgeneralizations regarding the relationship between the type of variable and the type of graph	-	-	-	√	√	√

UC: University classroom environment

RC: Real middle school classroom environment

The pre-service teachers designed the tasks and activities based on the idea that*"quantitative (numerical) variables are represented with a line graph and categorical variables are represented with a pie or a bar graph."* When designing the lesson for the first objective, they mainly took into account the type of the variable. They know about variables (categorical or quantitative (numerical)) and used this criterion to decide which type of graph to use. Since they did not formulate a question initially, they disregarded the purpose of displaying data with graphs. Even though taking into account the type of variable is not false while deciding the most suitable type of graph, the pre-service teachers overlooked that some variables can be measured as categorical or quantitative (numerical) depending on the question. For example, a quantitative variable can be converted into a categorical variable depending on the question formulated by the researcher. Thus, while deciding the suitable type of graph, the type of the variable should be evaluated within the context of the question. This lack of knowledge on the part of the pre-service teachers resulted in their inability to answer the questions asked by students during the implementation. For example, one of the students asked "*Can we convert all the bar graphs into pie* graph*s?" (University Implementation-Lesson study 1)*. Gamze stated that because the variable given [degrees of temperatures] is continuous, using pie graph would not be suitable. Here she overlooked the fact that depending on the question and purpose, we could measure or convert data [degrees of temperatures] into a suitable form and use a pie graph. During the reflection meeting, it was revealed that the pre-service teachers are of the opinion that a data set involving degrees of temperature should always be represented with a line graph because it is a continuous variable. Even though this issue was discussed, the group did not make any changes in their lesson plans and real classroom implementations.

With regard to the second objective, the pre-service teachers decided to introduce line graphs by making connections with bar graphs that students already knew. However, their conception about the relationship between variable type and graph type led them to focus on continuity of the variable and did not take the question or purpose into consideration. After the students had drawn the line graph, Şirin drew attention to the continuity of the variable. She stated that the bar graph is suitable for categorical variables and the line graph is suitable for quantitative (numerical) variables. However, her examples (representing the number of supporters for football teams with a bar graph; degrees of temperature in a day with a line graph) could have resulted in overgeneralization. She did not mention that the variables could be represented with different types of graph depending on the question or the purpose. This lack of understanding was observed in their responses to students' questions as well. For instance, when one of the students asked whether line graph is suitable to represent the precipitation rate, Şirin responded "*Yes, as it is* a *quantitative variable, it can be used."* In her response, she merely focused on the type of variable. After this instance was reminded and discussed in the evaluation meeting, they began to realize that while deciding on the type of graph, taking the variable type into account is not enough on its own. Şirin wrote in her reflection journal that *"I realized that I had been wrong about where to use the line graph. In this implementation, we learned that the line graph can be used with variables that are not continuous, that we need to decide depending on the question and that basically in situations where change can be observed, the line graph can be used*."

Related to the third objective in the real classroom implementation, Beyza asked students, "*Can I show the weight [of a group of people] with a bar graph?"* With this question, she intended to make students realize that weight can be represented with a bar graph when necessary. The pre-service teachers stated that students would overgeneralize weight as a continuous variable, and thus they felt the need to ask such a question. This instance also shows that the pre-service teachers considered the difficulties experienced by students and structured their implementations accordingly.

Understandings Related to the Graph Construction and Elements of the Graph

The development of pre-service teachers' understandings and their instructional practices related to the graph construction and elements of the graph are summarized in Table 6.

Table 6

Understandings Related to the Graph Construction and Elements of the Graph

	1st Lesson Study		2nd Lesson Study		3rd Lesson Study	
Criteria	UC	RC	UC	RC	UC	RC
Understanding and relating basic elements of different types of graphs	-	-	-	√	√	√
Attending to student difficulties in drawing graphs	-	-	-	√	√	√
Guiding students to draw graphs	-	-	-	√	√	√
Supporting students to think about and relate basic elements of different types of graphs	-	-	-	√	√	√

UC: University classroom environment

RC: Real middle school classroom environment

In general, the pre-service teachers were observed to know the basic elements constituting a graph (e.g., bar heights, axes, scaling, lines, pie sectors). This caused them to think that the students also knew these elements. Therefore, they either did not focus on or implicitly mentioned these elements. Over time they realized that they needed to talk about these elements more explicitly. For instance, during the implementation of real classrooms, Beyza observed that one group of students construct the graph without paying attention to equal scaling. The dialogue below shows her guidance to the group:

> Real Classroom Implementation-Lesson study 3
>
> Beyza: Could we directly put the values like this [values not at equal intervals]? You put the values at equal intervals [referring to leaving equal space among different values].
>
> Student: No, we can't.
>
> Beyza: Then, how should we do? Look. Here, they wrote 57, then 63. Then they would write 68 with leaving the same interval between values. Can we do it like this? Then you will write here 127.
>
> …..
>
> Student: So, can we write it like 57, 67,87?
>
> Beyza: Yes, it can be. Why not. For example, you place the data at equal intervals *like 10, 20, 30... and then put the remaining data somewhere in-between them.*

Here, it is seen that Beyza asked questions that would lead students to think about how scaling should be done. We, however, did not observe an explanation or discussion about why equal scaling is important.

Implementations showed that pre-service teachers also struggled with understanding the basic elements of graphs in a given context. For instance, during the university implementation, Şirin asked which type of graph could be used to represent degrees of average temperature of a city for five days. Students stated that as the degrees of temperature may take negative values, a pie graph would not be suitable for representing these data. The dialogue below shows the class discussion.

University Implementation-Lesson study 2

Şirin: I will present average degrees of temperature of a city for five days. I want you to show this with a graph. Which graph do you think I should use?

….

Student: I think we cannot use a pie graph because it might take negative or positive values. But can we show this [referring to negative values] in a pie graph?

Şirin: Yes, a very good explanation. If there is a negative degree of temperature, using a pie graph seems to be a bit difficult.

In this example, Şirin seems to have experienced difficulty in understanding what a sector of a pie meant for this data set (i.e., degrees of temperature). She overlooked the fact that each pie sector represents the number of observations of each category (e.g., number of days with -5^{o}C), that is, the frequency. She tried to represent values (e.g., -5^{o}C) rather than frequencies with sector of a pie chart that leads her to reach an unreasonable conclusion. After the evaluation meeting in which this issue was brought up, the pre-service teachers emphasized what the whole of a pie graph and each sector represents in the real classroom implementation.

Understandings Related to Reading and Interpreting Graphs

The development of pre-service teachers' understandings and instructional practices related to the reading and interpreting graphs is summarized in Table 7.

Table 7

Understandings Related to Reading and Interpreting Graphs

	1st Lesson Study		2nd Lesson Study		3rd Lesson Study	
Criteria	UC	RC	UC	RC	UC	RC
Relating findings from graphical representations with formulated questions	-	-	-	-	√	√
Asking questions for the purpose of reading between data (i.e., compare and explore data within the graph)	√	√	√	√	√	√
Asking questions for the purpose of reading beyond data (e.g., making generalizations and predictions)	-	-	-	-	-	-

UC: University classroom environment

RC: Real middle school classroom environment

Pre-service teachers did not put much emphasis on reading tables and graphs. Even though Gamze pointed out that students may only focus on the points and ignore the values between them when reading the line graph, the group did not consider her idea when designing the lesson. Their understandings related to the interpretations of graphs mainly included reading between the data without taking into account the purpose of the questions. There were no instances observed related to reading beyond the data. For the first objective related to pie graphs, they planned to ask students questions to compare the categories/groups. Even though comparing categories presented in a pie graph is not incorrect, such interpretations could be made through bar graph as well. What makes pie graph distinct as a tool to represent data is that it allows us to see the relative contribution of each category within the whole data set. Below is presented the related section from the lesson environment.

> University Implementation-Lesson study 1
>
> Gamze: By looking at this graph, what can you tell about which person has the largest number of siblings, about numbers such as fewer or more? Look at them and try to interpret them.
>
> Student: For example, half of 20 is 10.There are 9 person having one sibling. The sum of two siblings and three siblings is also 9 persons. I added and found 18.
>
> ….
>
> Gamze: Correct, can you make comments such as this is more and that is less?
>
> Student: The number of those having one sibling is more than those of the others.

After the discussion at the reflection meeting, pre-service teachers decided to include explorations of the contributions of each category within the whole data set. In real classroom implementation, Gamze emphasized that some points that could not be clearly seen in the bar graph could be seen better in the pie graph or vice versa. In the third lesson study, the pre-service teachers focused more on interpretations of the graphs and made comments associated with the statistical question. For instance, when one student made a comment that would be more suitable for a bar graph instead of the pie graph, the pre-service teacher reminded the purpose of using pie graphs and make the student realize that the pie graph allows interpretation about the meaning of the related part within the whole.

Discussion and Conclusion

One of the important findings of the current study is that the pre-service teachers did not see the process of doing statistics as consisted of many inter-related components. In particular, they did not consider that doing statistics starts with formulating questions and that the question affects the subsequent processes such as collecting data, selecting types of data display and interpreting graphs. This lack of understanding caused them to design superficial and disconnected tasks and activities. For instance, when they addressed an objective related to drawing graphs, they merely focused on the procedural aspects (e.g., drawing it correctly). They did not consider issues such as which statistical questions were sought to be answered or how data would have been collected. The fact that the pre-service teachers did not put the process of making statistics into centre while structuring their lessons, and that they focused on operational issues such as creating graphs and making statistical calculations are parallel to the findings that have been reported and emphasized in the literature (Chick & Pierce, 2008; Garfield & Ben-Zvi, 2008; Heaton & Mickelson, 2002; Ijeh, 2012; Lee et al., 2014; Reston et al., 2006). It is stated that even experienced teachers structure their lessons by focusing on statistical calculations (Quintas et al., 2014). The prior experiences with learning and doing statistics that were not structured in a way to support the statistical process might have paved the way for the emergence of such results (Arı, 2010).

Teachers' and pre-service teachers' lack of understandings related to doing statistics have been widely emphasized by prior research (Burgess, 2007; Espinel et al., 2008; Hannigan et al., 2013; Koleza & Kontogianni, 2016; Sorto, 2004). A number of studies have found that pre-service teachers and teachers disregarded the context when working with data sets and deciding on the appropriate graph type and associated the data with operational procedures (Burgess, 2002; Chick & Pierce, 2008). Likewise, in the current study, pre-service teachers demonstrated some misunderstandings and overgeneralizations related to the role of context in doing statistics which impacted their use of graphs in doing statistics. Conceptions such as *"contexts involving continuous variables are represented with line graphs"* and *"data with low frequency is represented by bar graphs, data with high frequency is represented by pie graphs."* impacted their decisions and actions when they designed and implemented lessons. These findings are in agreement with Burgess's (2007) findings which showed that pre-service teachers, who did not have a learning experience where the context is in the nature of statistics and questions are formulated according to the context, see statistics as formulas and calculations, and take certain generalizations into account when making inferences (Burgess, 2007).

The pre-service teachers' initial ideas and understandings developed gradually as they designed lesson plans, discussed about the concepts and reflected on their lesson plans and instructional implementations. In particular, they realized that doing statistics starts with a question that can be answered by collecting and analyzing data; and the question affects all components of doing statistics. The pre-service teachers' starting to put formulating questions into the centre of doing statistics became an important turning point significantly affected their conceptions related to doing and teaching statistics. For instance, while they initially viewed graphs as tools to represent data differently; they began to view graphs as tools to answer a statistical question. Such an understanding enabled them to compare and contrast different types of graphs in relation to answering particular questions. As a result, while deciding on the type of graph suitable for a data set, they took into account not only the type of the variable but also the questions addressed. Likewise, they began to think about the context of the tasks in relation with a statistical question addressed by collecting and analyzing data. Pre-service teachers also found opportunities to challenge their misunderstandings and overgeneralizations related to the role of context and type of variable as they designed and implemented the lessons. Also, the aspects of doing statistics that they initially did not consider but observed in students' work triggered the expansion of their understandings and teaching practices.

Throughout the study, we observed several instances that they transformed their understandings into teaching practices. For instance, with the understanding of the role formulated questions play in doing statistics, pre-service teachers began to evaluate tasks and context of the tasks in relation to a problem situation, a purpose or a question. They also revised their utterances (e.g., explanations, questions) in lesson plans and instructional implementations to emphasize the role of question in doing statistics. They began to create opportunities for students to think about the problem and the question presented in a statistical situation. In addition, they brought the role of graphs as tools to answer a statistical problem to the fore and asked questions to make students think in this direction. During the study, the pre-service teachers also realized that data collection is a part of doing statistics and needs to be included in the instruction. Asking students questions about how data could have been collected could be considered as an example of transforming their understanding into their teaching practices. As they became more aware of the importance of the data collection process, they began to pay greater attention to the use of real-life data, which could improve statistical thinking (Garfield & Everson, 2009).

The changes observed in the pre-service teachers' understandings and their teaching practices could be attributed to several factors enabled by the lesson study that they participated. The literature has been pointed out that pre-service teachers' working with students is an effective tool to foster their development (Ball & Cohen, 1999; Ball & Forzani, 2009). The questions asked, explanations made and responses given by the students during the instructional implementations were found to have helped the pre-service teachers transform their understandings into teaching practices. Implementations provide guidance to the pre-service teachers about the points that students might have difficulty with (Ijeh, 2012). In this study, the participants began to consider students' thinking when selecting tasks and structuring the instructional moves. For instance, when choosing the context of the task, they considered situations that may cause over-generalizations (e.g., votes are represented by pie graph). They began to take into account possible student reactions, conceptions, or mistakes. Experiences in real classroom implementations, especially unexpected situations, emerged during the implementations, and discussions and reflections on these experiences enabled them to reconsider their assumptions about students' thinking.

Research suggests that classroom activities should be designed in such a way as to put a great emphasis on the process of doing statistics (Garfield & Everson, 2009; Green & Blankenship, 2013; Heaton & Mickelson, 2002; Visnovska & Cobb, 2019). In the current study, a discussion was conducted with the pre-service teachers about the components of doing statistics before the lesson study. Yet, it was observed from the initial implementations that such a discussion was not very meaningful for them. When they started to work on instructional practices, the discussions conducted at the beginning became more meaningful to them. When they designed lesson plans, conducted instructional implementations and talked about these practices, they became personally involved in the process and gained a greater awareness of doing statistics. Namely, their personally experiencing of the process by means of designing and conducting instructional implementations in both the university and real classroom environments can be seen as an important key point supporting the development of the pre-service teachers' understandings and teaching practices.

Evaluation of the lessons is emphasized to be one of the important factors allowing teachers and pre-service teachers to see the deficiencies in their lessons and compensate for them (Hiebert & Morris, 2012; Santagata et al., 2007). In this study, the pre-service teachers' discussion in groups and discussions conducted in the university and real classroom environments after their presentations to overcome the problems they experienced can be seen as other key points. These discussions facilitated the pre-service teachers to take different perspectives and allowed them to share information as well. For example, the question asked by the expert, “*Why do you draw a pie graph? You need to think about this*.” encouraged the pre-service teachers to think about their knowledge related to the formulation of questions. Thus, such experiences can be seen as another important factor triggering information exchange among them (Guskey, 2003; Hiebert et al., 2003) in developing understandings and teaching practices.

When all the lesson study cycles are evaluated together, it is seen that though the changes mentioned above were observed in the pre-service teachers’ understandings, they still demonstrated difficulties in producing effective lessons for teaching graphs. For instance, they were observed to have difficulties designing tasks with meaningful problem situations or statistical questions, associating the data collection and interpretation of the results with the questions formulated. These findings support literature that the transition of subject knowledge into pedagogical content knowledge requires considerable time and effort and changes are not immediate (Friel & Bright, 1998; Fullan, 1991). It can be said that throughout the lesson study process, the authors did not present an instruction or intervene in planning activities. The pre-service teachers received feedback only after the implementation of the lesson plans in the university and in the real classroom settings. A more structured lesson study process along with an instruction focused on teaching graphs can make a greater contribution to the development of knowledge. Due to the difficulties involved in planning both the university and real classroom applications, the number of the participants was kept limited, which is another limitation of the current study. In addition, limited physical and technological conditions in real classroom settings did not allow the inclusion of technology in the instructional implementations. This can also be seen as another limitation of the study.

It can be thought that this study contributes to the literature by revealing the development of pre-service teachers’ understandings of doing statistics. In this regard, the current study is believed to guide researchers in developing content for both in-service and pre-service teacher training programs. The results of this study indicate that doing statistics should be put into the center of the content of the courses. Moreover, the programs should allow pre-service teachers to carry out activities and practices to make sense of this process. Components of lesson study such as planning, implementing, revising and evaluating contributed to the development of the pre-service teachers’ understandings. In particular, implications in university classrooms and real middle school classrooms support learning in theory and practice. At this point, attention should be paid to school-university cooperation and curriculum developers are suggested to design courses that will allow pre-service teachers to practice in real school environments.

Statement of Responsibility

Nadide Yılmaz; conceptualization, methodology, data collection validation, investigation, resources, data collection, design of research process, writing - original draft, writing - review & editing. İ. Elif Yetkin Özdemir; conceptualization, methodology, validation, writing - original draft, writing - review & editing, visualization, supervisior.

Conflicts of Interest

There is no conflict of interests for this study.

References

Ader, E. (2018). Programlardaki veri ve olasılık öğrenme alanı içeriklerine karşılaştırmalı bir bakış [A comparative look at data and probability learning area contents in programs]. In M. F. Özmantar, H. Akkoç, B. Kuşdemir Kayıran, & M. Özyurt (Eds.), *Ortaokul matematik öğretim programları: Tarihsel bir inceleme* (pp. 275-306). Pegem Akademi.

Arı, E. (2010). *İlköğretim 6-7 ve 8. sınıflarda matematik dersinin istatistik ve olasılık konusunun öğreniminde yaşanan problemler ve çözüm önerileri* [Yüksek lisans tezi]. Afyon Kocatepe Üniversitesi.

Arnold, P. (2008). Developing new statistical content knowledge with secondary school mathematics teachers. In C. Batanero, G. Burrill, C. Reading, and A. Rossman (Eds.), *Joint ICMI/IASE Study: Teaching Statistics in School Mathematics. Challenges for Teaching and Teacher Education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference*, ICMI and IASE. Springer.

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: toward practice-based theory of professional education. In L. Darling-Hammond,& G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3-32). CA: Jossey-Bass.

Ball, D. L., & Forzani, F. (2009). The work of teaching and challenge for teacher education. *Journal of Teacher Education, 60*(5), 497-511. https://doi.org/10.1177/0022487109348479

Bargagliotti, A., Franklin, C., Arnold, P., Gould, R., Johnson, S., Perez, L., & Spangler, D. (2020). *Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education (GAISE) report II*. American Statistical Association and National Council of Teachers of Mathematics.

Batanero, C., Arteaga, P., & Ruiz, B. (2010). Statistical graphs produced by prospective teachers in comparing two distributions. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education*. Lyon: ERME. Retrieved from www.inrp.fr/editions/editions-electroniques/cerme6/

Batanero, C., Burrill, G. F., & Reading, C. (Eds.). (2011). *Teaching statistics in school mathematics—challenges for teaching and teacher education: A Joint ICMI/IASE Study: The 18th ICMI Study*. Springer.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher, 33*(8), 3-15. https://doi.org/10.3102/0013189X033008003

Burgess, T. A. (2001). Assessing the statistics knowledge of pre-service teachers. In J. Bobis, B. Perry, and M. Mitchelmore (Eds.), *Numeracy and beyond (Proceedings of the 24th annual conference of the Mathematics Education Research Group of Australasia)*, (pp. 114-121). MERGA.

Burgess, T. A. (2002). Investigating the 'data sense' of preservice teachers. In B. Phillips (Ed.), *Proceedings of the Sixth International Conference on Teaching*

Statistics.: International Statistical Institute and International Association for Statistics Education.

Burgess, T. A. (2007). *Investigating the nature of teacher knowledge needed and used in teaching statistics* [Unpublished doctoral dissertation]. Massey University.

Carver, R., Everson, M., Gabrosek, J., Horton, N., Lock, R., Mocko, M., Rossman, A., Rowell, G. H., Velleman, P., Witmer, J., & Wood, B. (2016). *Guidelines for assessment and instruction in statistics education: College Report 2016.* American Statistical Association.

Chick, H. L., & Pierce, R. U. (2008). Teaching statistics at the primary school level: Beliefs, affordances, and pedagogical content knowledge. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Joint ICMI/IASE study: Teaching statistics in school mathematics. Challenges for teaching and teacher education. Proceedings of the ICMI Study 18 and 2008IASE Round Table Conference.* Monterrey: ICMI and IASE. Erişim adresi: http://www.ugr.es/-icmi/iase_study

Cohan, A., & Honigsfeld, A. (2006). Incorporating lesson study in teacher preparation. *The Educational Forum, 71*(1), 81-92. https://doi.org/10.1080/00131720608984570

Curcio, F. R. (1987). Comprehension of mathematical relationships expressed in graphs. *Journal for Research in Mathematics Education, 18*(5), 382-393. https://www.jstor.org/stable/749086

delMas, R. C. (2004). A comparison of mathematical and statistical reasoning. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning, and thinking* (pp. 79-95). Kluwer.

Eicher, A., & Zapata-Cardana, L. (2016). *Empirical research in statistics education*, Springer.

English, L. D. (2014). Statistics at play. *Teaching Children Mathematics, 21*(1), 36-44. https://www.jstor.org/stable/10.5951/teacchilmath.21.1.0036

English, L. D., Watson, J., & Fitzallen, N. (2017). Fourth-graders' meta-questioning in statistical investigations. In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th annual conference of the Mathematics Education Research Group of Australasia*, (pp. 229-236). MERGA.

Espinel, M. C., Bruno, A., & Plasencia, I. (2008). Statistical graphs in the training of teachers. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Joint ICMI/IASE study: Teaching statistics in school mathematics. Challenges for teaching and teacher education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference.* Monterrey: ICMI and IASE. Springer.

Fernandez, M. L. (2005). Learning through microteaching lesson study in teacher preparation. Action *in Teacher Education, 26*(4), 37-47. https://doi.org/10.1080/01626620.2005.10463341

Franklin, C., Bargagliotti, A. E., Case, C. A., Kader, G. D., Scheaffer, R. L., & Spangler, D. A. (2015). *Statistical education of teachers (SET).* VA: American Statistical Association.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2005) *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K–12 Curriculum Framework.* American Statistical Association.

Friel, S. N., & Bright, G. W. (1998). Teach-Stat: A model for professional development in data analysis and statistics for teachers K-6. In S.P. Lajoie (Ed.), *Reflections on St Romberg statistics* (pp. 63-117). Lawrence Erlbaum.

Friel, S. N., Curcio, R. F., & Bright, G. W. (2001). Making sense of graphs: Critical factors influencing comprehension and instructional implications. *Journal for Research in Mathematics Education, 32*(2), 124-158. https://doi.org/10.2307/749671

Fullan, M. G. (1991). *The new meaning of educational change*. Cassell Educational Limited Wellington House.

Garfield, J., & Ben-Zvi, D. (2004). Research on statistical literacy, reasoning, and thinking: issues, challenges, and implications. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning, and thinking* (pp. 397-409). Kluwer Academic Publishers.

Garfield, J., & Ben-Zvi, D. (2008). Preparing school teachers to develop students' statistical reasoning, In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.) *Teaching Statistics in School Mathematics-Challenges for Teaching and Teacher Education, A* Joint ICMI/IASE Study: The 18th ICMI study (pp.187-198). Springer.

Garfield, J., & Everson, M. (2009). Preparing teachers of statistics: A graduate course for future teachers. *Journal of Statistics Education, 17*(2), 1-16. https://doi.org/10.1080/10691898.2009.11889516

Graham, A. (2006). *Developing thinking in statistics*. CA: Sage Publications.

Green, J., & Blankenship, E. E. (2013). Primarily statistics: Developing an introductory statistics course for pre-service elementary teachers. *Journal of Statistics Education*, *21*(3), 1-21. https://doi.org/10.1080/10691898.2013.11889683

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. W. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record*, *111*(9), 2055-2100.

Groth, R. E. (2007). Toward a conceptualization of statistical knowledge for teaching. *Journal for Research in Mathematics Education, 38*(5), 427-437. https://www.jstor.org/stable/30034960

Gürel, R. (2016). *An investigation of middle school mathematics teachers' knowledge for teaching measures of central tendency and variation* [Unpublished doctoral thesis]. Hacettepe Üniversitesi.

Guskey, T. R. (2003). What makes professional development effective? *Phi Delta Kappan, 84*(10), 748-750. https://doi.org/10.1177/003172170308401007

Güven, B., Öztürk, T., & Özmen, Z. M. (2015). Examining the statistical process experiences of 8th grade students. *Education and Science, 40*(177), 343-363. http://dx.doi.org/10.15390/EB.2015.3313

Hacısalihoğlu-Karadeniz, M. (2016). Determining the data processing learning domain attainments of 5th grade students. *Mediterranean Journal of Humanities*, *6*(1), 221-236. http://dx.doi.org/10.13114/MJH.2016119300

Hannigan, A., Gill, O., & Leavy, A. M. (2013). An investigation of prospective secondary mathematics teachers' conceptual knowledge of and attitudes towards statistics. *Journal of Mathematics Teacher Education, 16*(6), 427-449. https://doi.org/10.1007/s10857-013-9246-3

Heaton, R. M., & Mickelson, W. T. (2002). The learning and teaching of statistical investigations teaching and teacher education. *Journal of Mathematics Teacher Education*, *5*(1), 35-59. https://doi.org/10.1023/A:1013886730487

Hiebert, J., & Morris, A. K. (2012). Teaching, rather than teachers, as a path toward improving classroom instruction. *Journal of Teacher Education*, *63*, 92-102. https://doi.org/10.1177/0022487111428328

Hiebert, J., Morris, A. K., Berk, D., & Jansen, A. (2007). Preparing teachers to learn from teaching. *Journal of Teacher Education, 58,* 47-61. https://doi.org/10.1177/0022487106295726

Hiebert, J., Morris, A. K., & Glass, B. (2003). Learning to learn to teach: An “experiment” model for teaching and teaching preparation in mathematics. *Journal of Mathematics Teacher Education, 6,* 201-222. https://doi.org/10.1023/A:1025162108648

Ijeh, S. B. (2012). *How competent mathematics teachers develop pedagogical content knowledge in statistics teaching* [Unpublished doctoral dissertation]. University of Pretoria.

Koleza, E., & Kontogianni, A. (2016). Statistics in primary education in Greece: How ready are primary teachers?. D. Ben-Zvi,& K. Makar (Ed.), *The Teaching and Learning of Statistics International Perspectives* (p. 289-298). Springer.

Kurt, G. (2015). An investigation of understanding of preservice elementary mathematics teachers (PEMT) about data displays. In K. Krainer & N. Vondrova ́ (Ed.), *Ninth Congress of the European Society for Research in Mathematics Education (CERME 9*), (pp.615-621). Prague, Czech Republic.

Leavy, A. (2006). Using data comparison to support a focus on distribution: Examining preservice teachers’ understandings of distribution when engaged in statistical inquiry. *Statistics Education Research Journal, 5*(2), 89-114.

Lee, H. S., Kersaint, G., Harper, S. R., Driskell, S. O., Jones, D. L., Leatham, K. R., Angotti, R. L., & Adu-Gyamfi, K. (2014). Teachers' use of transnumeration in solving statistical tasks with dynamic statistical software. *Statistics Education Research Journal*, *13*(1), 25-52.

MacGillivray, H., & Pereira-Mendoza, L. (2011). Teaching Statistical Thinking ThroughInvestigative Projects. C. Batanero, G. Burrill, and C. Reading (Eds.), *Teaching statistics in schoolmathematics-challenges for teaching and teacher education: A joint ICMI/IASE study* (p.109-120). Springer.

Makar, K., & Fielding-Wells, J. (2011). Teaching teachers to teach statistical investigations. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics – challenges for teaching and teacher education: A joint ICMI/IASE study* (pp. 347-358). Springer.

Mercimek, O. (2013). *Assessment of preservice mathematics teachers’ knowledge for teaching statistics* [Doctoral dissertation]. Middle East Technical University.

Meyer, R., & Wilkerson, T. (2011). Lesson study: The ımpact on Teachers’ knowledge for teaching Mathematics, In Hardt, L., Alston, A., Yoshido, M., & Murata, A.

(Eds.) *Lesson study and practice: Lesson study Research and Practice in Mathematics Education: Learning Together* (pp.15-26). NY: Springer.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded Sourcebook* (2nd ed). Thousand Oaks, CA: Sage

Ministry of National Education [MoNE]. (2009). *İlköğretim matematik dersi 6-8. sınıflar öğretim programı* [Elementary school mathematics curricula for grades 6-8]. Milli Eğitim Yayınevi.

Ministry of National Education [MoNE]. (2013). *Ortaokul matematik dersi (5, 6, 7, ve 8. sınıf) öğretim programı* [Middle school mathematics (5, 6, 7, and 8th grades) curricula]. Milli Eğitim Yayınevi.

Ministry of National Education [MoNE]. (2018). *Matematik dersi (İlkokul ve ortaokul 1, 2, 3, 4, 5, 6, 7 ve 8. sınıflar) öğretim programı* [Primary and middle school mathematics curricula for grades 1, 2, 3, 4, 5, 6, 7, and 8]. Milli Eğitim Yayınevi.

Moore, D. S. (1988). Should mathematicians teach statistics?. *The College Mathematics Journal*. *19*(1), 3-7. https://www.jstor.org/stable/2686687

Morris, A. K., Hiebert, J., & Spitzer, S. M. (2009). Mathematical knowledge for teaching in planning and evaluating instruction: What can pre-service teachers learn?. *Journal for Research in Mathematics Education, 40*(5), 491-529.

Murata, A. (2010). Teacher learning with lesson study, In P. Peterson, E. Baker, & B. McGaw (Ed.), *International Encyclopedia of Education, 7* (pp. 575-581). Oxford: Elsevier.

National Council of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics.* VA: Author.

Öz, Ö. (2019). *An investigation of 7th grade students' level of understanding in the statistical investigation process* [Unpublished master thesis]. Middle East Technical University.

Pfannkuch, M. (2005). Characterizing year 11 students' evaluation of a statistical process. *Statistics Education Research Journal, 4*(2), 5-26.

Quintas, S., Tomás Ferreira, R., & Oliveira, H. (2014). Attending to students' thinking on bivariate statistical data at secondary level: Two teachers' pedagogical content knowledge. In K. Makar, B de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the 9th International Conference on Teaching Statistics* (ICOTS9). Flagstaff, Arizona, USA: Voorburg, The Netherlands: International Statistics Institute.

Reston, E., Jala, L., & Edullantes, T. (2006). Probing college statistics teachers' instructional goals and classroom practices within a statistical literacy framework. In A. Rossman & B. Chance (Eds.), *Working cooperatively in statistics education: Proceedings of the 7th International Conference on Teaching Statistics, Salvador, Brazil*. International Association for Statistical Education and International Statistical Institute: Voorburg, The Netherlands.

Rossman, A., Chance, B., & Medina, E. (2006). Some important comparisons between statistics and mathematics and why teachers should care. In G. F. Burrill & P. C. Elliot (Eds.), *Thinking and reasoning with data and chance* (pp. 323-333). VA: NCTM.

Santagata, R., Zannoni, C., & Stigler, J. W. (2007). The role lesson analysis in pre-service teacher education. An empirical investigation of teacher learning from a virtual video-based field experience. *Journal of Mathematics Teacher Education, 10*, 123-140. https://doi.org/10.1007/s10857-007-9029-9

Santos, R., & Ponte, J. P. (2014). Learning and teaching statistical investigations: A case study of a prospective teacher. In K. Makar, B. de Sousa, & R. Gould (Ed.), *Sustainability in statistics education. Proceedings of the 9th International Conference on Teaching Statistics, ICOTS9, July, 2014, Flagstaff, AZ, USA*. International Statistical Institute: Voorburg, The Netherlands.

Schwartz, J. E. (2008). *Elementary mathematics pedagogical content knowledge: Powerful ideas for teachers*. Pearson.

Shaughnessy, J. M. (2007). Research on statistics learning and reasoning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning. A project of the National Council of Teachers of Mathematics* (pp. 957-1009). NC: Information Age.

Sorto, M. A. (2004). *Prospective middle school teachers' knowledge about data analysis and its application to teaching* [Unpublished doctoral dissertation]. Michigan State University.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2010). *Elementary and middle school mathematics: Teaching developmentally*. Pearson Education.

Visnovska, J., & Cobb, P. (2019). Supporting shifts in teachers' views of a classroom statistical activity: Problem context in teaching statistics. *Mathematical Thinking and Learning, 21*(4), 285-305. https://doi.org/10.1080/10986065.2019.1576003

Wright, T. D. (2009). *Investigating teachers' perspectives on the impact of the lesson study process on their mathematical content knowledge, pedagogical knowledge, and the potential for student achievement* [Unpublished doctoral dissertation]. University of New Orleans, Louisiana, United States.

Yamnitzky, G. S. (2010). *Elementary teachers' perspectives on the impact that Lesson Study participation had on their mathematical content and pedagogical content knowledge* [Unpublished doctoral dissertation]. University of Pittsburgh.

Zhang, S., & Cheng, Q. (2011). Learning to teach through a practicum-based microteaching model. *Action in Teacher Education, 33*(4), 343-358. https://doi.org/10.1080/01626620.2011.620523